5.2.1 Exercises

For a link to all of the additional resources available for this section, click OSttS Chapter 5 materials.

In Exercises 1 - 20, show that the given function is one-to-one and find its inverse. Check your answers algebraically and graphically. Verify that the range of f is the domain of f^{-1} and vice-versa. For help with these exercises, click one or more of the resources below:

- Identifying a one-to-one function using the Horizontal Line Test
- Finding the formula for an inverse function
- Verifying two functions are inverses using function composition
- Graphs of inverse functions

1. $f(x) = 6x - 2$
2. $f(x) = 42 - x$
3. $f(x) = \dfrac{x-2}{3} + 4$
4. $f(x) = 1 - \dfrac{4+3x}{5}$
5. $f(x) = \sqrt{3x-1} + 5$
6. $f(x) = 2 - \sqrt{x-5}$
7. $f(x) = 3\sqrt{x-1} - 4$
8. $f(x) = 1 - 2\sqrt{2x+5}$
9. $f(x) = \sqrt[5]{3x-1}$
10. $f(x) = 3 - \sqrt[3]{x-2}$
11. $f(x) = x^2 - 10x,\ x \geq 5$
12. $f(x) = 3(x+4)^2 - 5,\ x \leq -4$
13. $f(x) = x^2 - 6x + 5,\ x \leq 3$
14. $f(x) = 4x^2 + 4x + 1,\ x < -1$
15. $f(x) = \dfrac{3}{4-x}$
16. $f(x) = \dfrac{x}{1-3x}$
17. $f(x) = \dfrac{2x-1}{3x+4}$
18. $f(x) = \dfrac{4x+2}{3x-6}$
19. $f(x) = \dfrac{-3x-2}{x+3}$
20. $f(x) = \dfrac{x-2}{2x-1}$

With help from your classmates, find the inverses of the functions in Exercises 21 - 24.

21. $f(x) = ax + b,\ a \neq 0$
22. $f(x) = a\sqrt{x-h} + k,\ a \neq 0, x \geq h$
23. $f(x) = ax^2 + bx + c$ where $a \neq 0$, $x \geq -\dfrac{b}{2a}$.
24. $f(x) = \dfrac{ax+b}{cx+d}$, (See Exercise 33 below.)

25. In Example 1.5.3, the price of a dOpi media player, in dollars per dOpi, is given as a function of the weekly sales x according to the formula $p(x) = 450 - 15x$ for $0 \leq x \leq 30$.

    (a) Find $p^{-1}(x)$ and state its domain.

    (b) Find and interpret $p^{-1}(105)$.

    (c) In Example 1.5.3, we determined that the profit (in dollars) made from producing and selling x dOpis per week is $P(x) = -15x^2 + 350x - 2000$, for $0 \leq x \leq 30$. Find $\left(P \circ p^{-1}\right)(x)$ and determine what price per dOpi would yield the maximum profit. What is the maximum profit? How many dOpis need to be produced and sold to achieve the maximum profit?

26. Show that the Fahrenheit to Celsius conversion function found in Exercise 35 in Section 2.1 is invertible and that its inverse is the Celsius to Fahrenheit conversion function.

27. Analytically show that the function $f(x) = x^3 + 3x + 1$ is one-to-one. Since finding a formula for its inverse is beyond the scope of this textbook, use Theorem 5.2 to help you compute $f^{-1}(1)$, $f^{-1}(5)$, and $f^{-1}(-3)$.

28. Let $f(x) = \frac{2x}{x^2-1}$. Using the techniques in Section 4.2, graph $y = f(x)$. Verify that f is one-to-one on the interval $(-1, 1)$. Use the procedure outlined on Page 401 and your graphing calculator to find the formula for $f^{-1}(x)$. Note that since $f(0) = 0$, it should be the case that $f^{-1}(0) = 0$. What goes wrong when you attempt to substitute $x = 0$ into $f^{-1}(x)$? Discuss with your classmates how this problem arose and possible remedies.

29. With the help of your classmates, explain why a function which is either strictly increasing or strictly decreasing on its entire domain would have to be one-to-one, hence invertible.

30. If f is odd and invertible, prove that f^{-1} is also odd.

31. Let f and g be invertible functions. With the help of your classmates show that $(f \circ g)$ is one-to-one, hence invertible, and that $(f \circ g)^{-1}(x) = (g^{-1} \circ f^{-1})(x)$.

32. What graphical feature must a function f possess for it to be its own inverse?

33. What conditions must you place on the values of a, b, c and d in Exercise 24 in order to guarantee that the function is invertible?

Checkpoint Quiz 5.2

1. Let $f(x) = \dfrac{3x}{x+1}$.

   (a) Assuming f is one-to-one, find a formula for $f^{-1}(x)$.

   (b) Check your answer to part (a) using function composition.

   (c) Find the range of f.

2. Explain why a function with more than one x-intercept cannot be invertible.

For worked out solutions to this quiz, click the link below:

- Quiz Solution

5.2.2 ANSWERS

1. $f^{-1}(x) = \dfrac{x+2}{6}$
2. $f^{-1}(x) = 42 - x$
3. $f^{-1}(x) = 3x - 10$
4. $f^{-1}(x) = -\frac{5}{3}x + \frac{1}{3}$
5. $f^{-1}(x) = \frac{1}{3}(x-5)^2 + \frac{1}{3},\ x \geq 5$
6. $f^{-1}(x) = (x-2)^2 + 5,\ x \leq 2$
7. $f^{-1}(x) = \frac{1}{9}(x+4)^2 + 1,\ x \geq -4$
8. $f^{-1}(x) = \frac{1}{8}(x-1)^2 - \frac{5}{2},\ x \leq 1$
9. $f^{-1}(x) = \frac{1}{3}x^5 + \frac{1}{3}$
10. $f^{-1}(x) = -(x-3)^3 + 2$
11. $f^{-1}(x) = 5 + \sqrt{x+25}$
12. $f^{-1}(x) = -\sqrt{\frac{x+5}{3}} - 4$
13. $f^{-1}(x) = 3 - \sqrt{x+4}$
14. $f^{-1}(x) = -\frac{\sqrt{x}+1}{2},\ x > 1$
15. $f^{-1}(x) = \dfrac{4x-3}{x}$
16. $f^{-1}(x) = \dfrac{x}{3x+1}$
17. $f^{-1}(x) = \dfrac{4x+1}{2-3x}$
18. $f^{-1}(x) = \dfrac{6x+2}{3x-4}$
19. $f^{-1}(x) = \dfrac{-3x-2}{x+3}$
20. $f^{-1}(x) = \dfrac{x-2}{2x-1}$

25. (a) $p^{-1}(x) = \frac{450-x}{15}$. The domain of p^{-1} is the range of p which is $[0, 450]$
    (b) $p^{-1}(105) = 23$. This means that if the price is set to \$105 then 23 dOpis will be sold.
    (c) $\left(P \circ p^{-1}\right)(x) = -\frac{1}{15}x^2 + \frac{110}{3}x - 5000,\ 0 \leq x \leq 450$. The graph of $y = \left(P \circ p^{-1}\right)(x)$ is a parabola opening downwards with vertex $\left(275, \frac{125}{3}\right) \approx (275, 41.67)$. This means that the maximum profit is a whopping \$41.67 when the price per dOpi is set to \$275. At this price, we can produce and sell $p^{-1}(275) = 11.\overline{6}$ dOpis. Since we cannot sell part of a system, we need to adjust the price to sell either 11 dOpis or 12 dOpis. We find $p(11) = 285$ and $p(12) = 270$, which means we set the price per dOpi at either \$285 or \$270, respectively. The profits at these prices are $\left(P \circ p^{-1}\right)(285) = 35$ and $\left(P \circ p^{-1}\right)(270) = 40$, so it looks as if the maximum profit is \$40 and it is made by producing and selling 12 dOpis a week at a price of \$270 per dOpi.

27. Given that $f(0) = 1$, we have $f^{-1}(1) = 0$. Similarly $f^{-1}(5) = 1$ and $f^{-1}(-3) = -1$